Year 9 - Half term 4 + 5
Challenge 9

Name	
Form	

Contents

Instructions:

- Complete the problems.
- Aim for at least 2 problems per session.
- I will be checking in to see how many problems you can solve!
- Ensure to display your work neatly in a book. Write the problem number and then your working.

Half Term 5

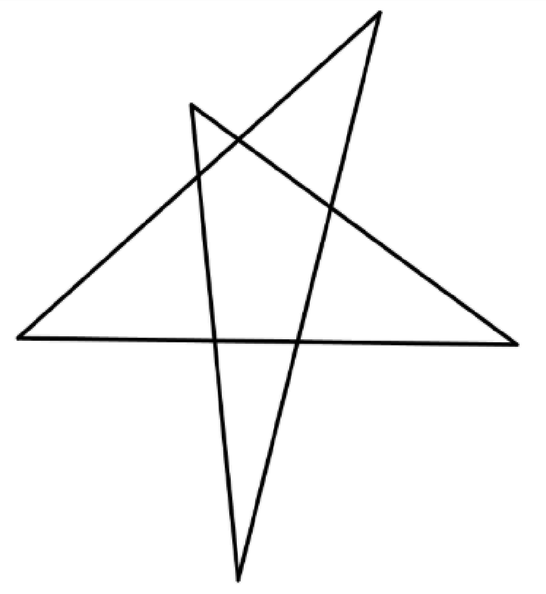
- In half term 6 you will spend some sessions completing the maths challenge team challenge! This is a national competition where teams from different schools compete against each other. We will do the same - you will divide into teams and complete the challenge together. I will be checking to see how many points you can get! (45 mins)

2025 Maths challenge
45 mins group round
https://ukmt.org.uk/wp-content/uploads/2025/06/TMC-2025-Regional-Group-Round-1.pdf#

Cross number
https://ukmt.org.uk/wp-content/uploads/2026/01/TMC-2025-Regional-Crossnumber-Round-2.pdf

Problem 1 (https://nrich.maths.org/problems/star-polygons?tab=overview)

Here is a five-pointed star:

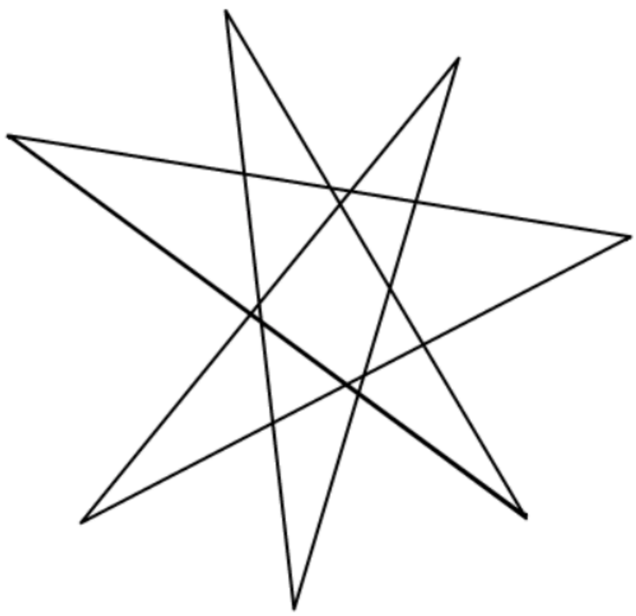


Draw a couple of five pointed stars of your own, making sure your lines are nice and straight!

Measure the interior angles at the five points and add them together.

What do you notice?

Here is a seven-pointed star:



Draw some seven-pointed stars of your own.

Measure the interior angles at the seven points and add them together.

What do you notice? Can you explain your findings?

Problem 2:

https://nrich.maths.org/problems/product-sudoku

A Short Demonstration

The cell in the top left corner of this Sudoku contains the number 20. 20 is the product of the digits in the two adjacent cells, which therefore must contain the digits 4 and 5. The 5 cannot go in the cell below the top left hand corner because 5 is not a factor of 96 (the product shown in the third cell down on the left hand side of the puzzle). Therefore 5 must be entered into the cell to the right of the cell containing 20 and 4 in the cell below.

By Henry Kwok

20					30			
96		324						
					405			3
180								
				600		36		
		2835						
4								

Rules of Product Sudoku

Like a conventional Sudoku, this Product Sudoku has two basic rules:

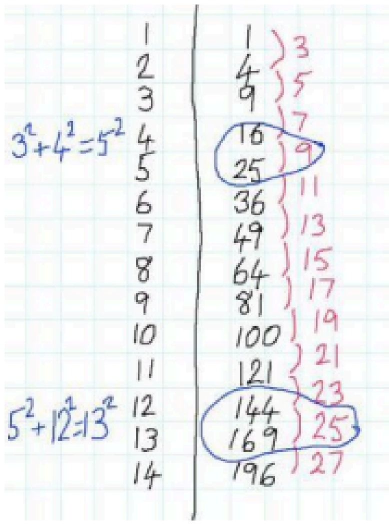
1. Each column, row and 3×3 subgrid must have the numbers 1 to 9.
2. No column, row or subgrid can have two cells with the same number.

The puzzle can be solved with the help of the numbers in the top parts of certain cells. These numbers are the products of the digits in all the cells horizontally and vertically adjacent to the cell.

Problem 3: https://nrich.maths.org/problems/generating-triples

Generating Triples

Charlie has been investigating square numbers. He decided to organise his work in a table:



Charlie noticed some special relationships between certain square numbers:

$3^2 + 4^2 = 5^2$ $\quad\quad$ $5^2 + 12^2 = 13^2$

Sets of integers like 3, 4, 5 and 5, 12, 13 are called **Pythagorean Triples**, because they could be the lengths of the sides of a right-angled triangle.

He wondered whether he could find any more...

Can you extend Charlie's table to find any more sets of Pythagorean Triples where the hypotenuse is 1 unit longer than one of the other sides?
Do you notice any patterns?
Can you make any predictions?

Can you find a formula that generates Pythagorean Triples like Charlie's?
Can you prove that your formula works?

Alison has been working on Pythagorean Triples where the hypotenuse is 2 units longer than one of the other sides.
So far, she has found these:

$4^2 + 3^2 = 5^2$ $\quad\quad$ $6^2 + 8^2 = 10^2$ $\quad\quad$ $8^2 + 15^2 = 17^2$

Some of these are just scaled-up versions of Charlie's triples, but some of them are new and can't be divided by a common factor (these are called **primitive triples**).

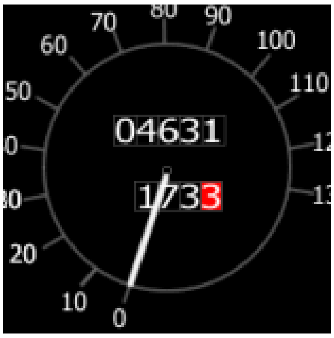
Can you find more Pythagorean Triples like Alison's?

Can you find a formula for generating Pythagorean Triples like Alison's?
Can you prove that your formula works?

Problem 4 https://nrich.maths.org/problems/how-many-miles-go

How Many Miles to Go?

A car's milometer reads 4631 and the trip meter has 173.3 on it:

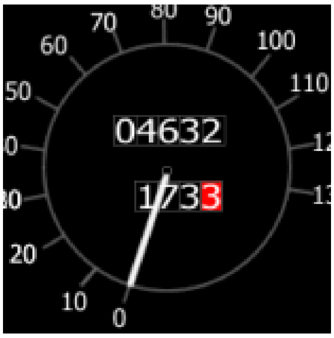


How many more miles must the car travel before the two numbers contain the same digits in the same order?

Can you find some other starting numbers that will lead to matching pairs?

What is the relationship between two starting numbers that will work?

If the milometer and trip meters had the following exact readings (4632 and 173.3) then the two numbers will not contain the same digits in the same order before the milometer reaches 99,999 miles. Why is this?



If the milometer 'loops the clock' back to 00000 again at 100,000 miles and the trip meter loops the clock back to 000.0 at 1000 miles, will the two numbers ever contain the same digits in the same order?

Problem 5
https://nrich.maths.org/sites/default/files/thumbnails/content-id-6390-Elevenses.pdf

nrich

Elevenses

In the grid below, look for pairs of numbers that add up to a multiple of 11.

9	46	79	13
64	90	2	97
25	31	20	22
4	52	55	7

Are there any numbers that can only have one partner?
Are there any numbers that could have more than one partner?
What is special about numbers which have the same set of partners?

Can you find every possible pair?
How can you be sure you haven't missed any?

You may have solved the problem by looking at how close each number is to a multiple of 11...

Here is another grid.
This time, look for pairs that add up to a multiple of 13.

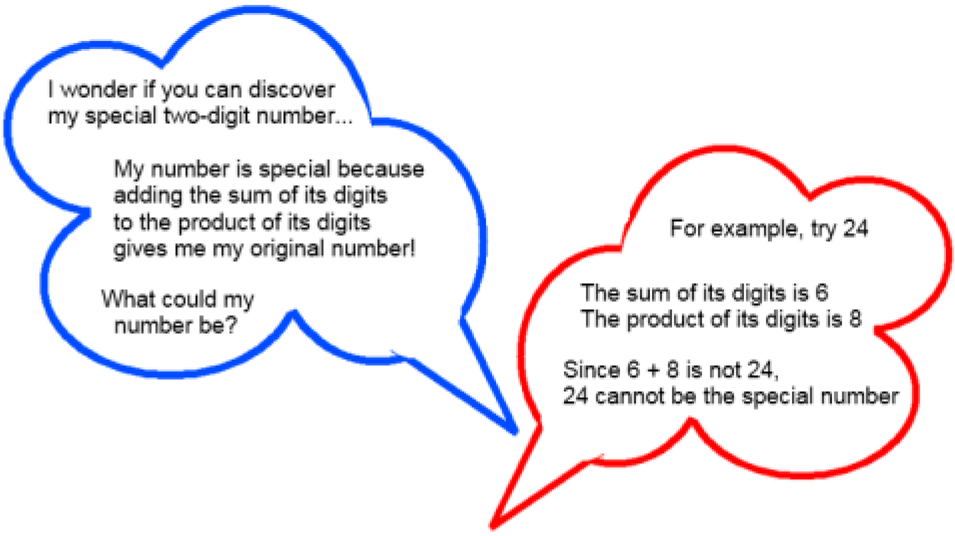
11	54	93	15
76	106	2	115
29	37	24	26
4	62	65	9

How can you use your insights from the first problem to be sure you have found all the possible pairings?

Problem 6 https://nrich.maths.org/problems/special-numbers

Special Numbers

One day our teacher asked us a puzzling question:



Can you find a special number?
Can you find more than one?
Can you find them all?

There are other sorts of special two-digit numbers...

- I add twice the tens digit to the units digit, then add this to the product of the digits. I get back to my original number.
- I add three times the tens digit to the units digit, then add this to the product of the digits. I get back to my original number.
- I add four times... or five times... or...

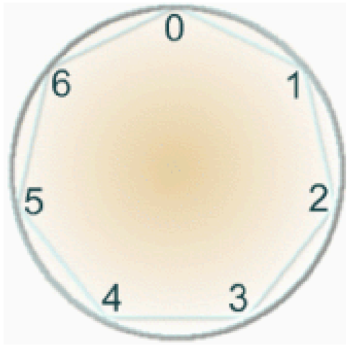
Can you use algebra to help you to find these special numbers?

Problem 7 https://nrich.maths.org/problems/days-and-dates

Days and Dates

If today is Monday we know that in 702 days' time (so in 100 weeks and 2 days' time) it will be Wednesday.

What day will it be in 15 days? 26 days? 234 days?



In 2, 9, 16 and 23 days from now, it will be a Wednesday.

What other numbers of days from now will be Wednesdays?
Can you generalise what you have noticed?

Choose a pair of numbers and find the remainders when you divide by 7. Then find the remainder when you divide the total by 7. For example:

15÷7 = 2 remainder 1	15+26 = 41
26÷7 = 3 remainder 5	41÷7 = 5 remainder 6

Choose some more pairs of numbers.
Is there a relationship between the remainders when you divide each by 7, and the remainder when you divide their total by 7.

Now find the remainder when you divide the **product** of 15 and 26 by 7. What happens?

Choose some more pairs of numbers.
Is there a relationship between the remainders when you divide each by 7, and the remainder when you divide their product by 7?

What about when you divide by numbers other than 7?
Can you explain what you've noticed?

How could you use these ideas to work out on which day of the week your birthday will be next year?

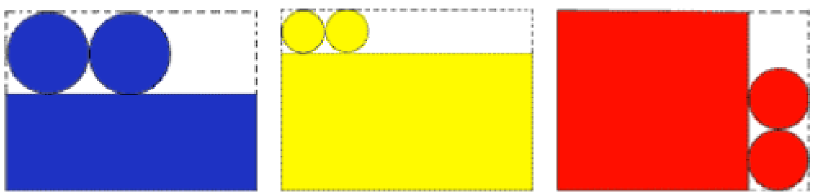
Problem 8 https://nrich.maths.org/problems/efficient-cutting

Efficient Cutting

Cylindrical containers, like the tin cans used to package some food, can be made using two circles for the ends, and a rectangle which wraps round to form the body.

To make cylinders of varying sizes, the three pieces can be cut from a single rectangle of flat sheet in several ways.

For example:



Can you work out some possible dimensions of a rectangle and two circles which can be cut from a single sheet of A4 paper and put together to make a cylinder?

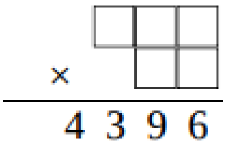
Which dimensions allow you to make a cylinder with the greatest volume?

You can assume that the dimensions of an A4 sheet of paper are 21cm and 29.6cm

Problem 9:
https://nrich.maths.org/sites/default/files/thumbnails/content-00-11-six3-FunnyFactorisation.pdf

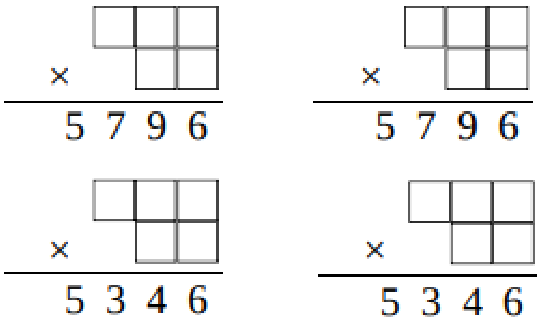
Funny Factorisation

Some 4 digit numbers can be written as the product of a 3 digit number and a 2 digit number using each of the digits 1 to 9 once, and only once.
The number 4396 can be written as just such a product.



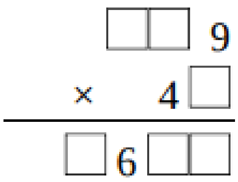
Can you find the factors?

Maths is full of surprises!
The numbers 5796 and 5346 can each be written as a product like this in two **different** ways.



Can you find these four funny factorisations?

Extension
There are two more funny factorisations to find, using each of the digits 1 to 9 once, and only once.
Can you fill in the blanks below to find one of them?



If you know a bit about computer programming, you may wish to write a program to find the final funny factorisation.

Problem 10: https://nrich.maths.org/problems/wipeout

Wipeout

Take the numbers 1, 2, 3, 4, 5, 6, and choose one to wipe out.

For example, you might wipe out 5, leaving you with 1, 2, 3, 4, 6.

The mean of what is left is 3.2

I wonder whether I can wipe out one number from 1 to 6, and leave behind an average which is a whole number...

What about starting with other sets of numbers from 1 to N, where N is even, **wiping out just one number**, and finding the mean?

Which numbers can be wiped out, so that the mean of what is left is a whole number? Can you explain why?

What happens when N is odd?

Here are some puzzling wipeouts you might like to try:

One of the numbers from 1, 2, 3, 4, 5, 6 is wiped out.
The mean of what is left is 3.6
Which number was crossed out?

One of the numbers from 1 to 15 is wiped out.
The mean of what is left is $7.\dot{7}1428\dot{5}$
Which number was crossed out?

One of the numbers from 1 to N, where N is unknown, is wiped out.
The mean of what is left is $6.8\dot{3}$
What is N, and which number was crossed out?

One of the numbers from 1 to N is wiped out.
The mean of what is left is 25.76
What is N, and which number was crossed out?

Maths Challenge! These questions increase in difficulty. How hard can you go?

Intermediate Mathematical Challenge — Wednesday 28 January 2026

1. Which of these has the largest value?

A $\frac{1+2}{2}$ B $\frac{1+2+3}{3}$ C $\frac{1+2+3+4}{4}$ D $\frac{1+2+3+4+5}{5}$ E $\frac{1+2+3+4+5+6}{6}$

2. What time will it be 2026 minutes after 20:26 on Monday?

A 06:12 on Tuesday B 16:12 on Tuesday C 20:12 on Tuesday D 06:12 on Wednesday E 20:12 on Wednesday

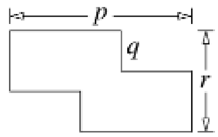
3. A cuboid has side-lengths, in cm, which are three consecutive positive integers. It has volume $120\,\text{cm}^3$. What is the area, in cm^2, of its largest face?

A 15 B 20 C 24 D 30 E 41

4. When 35 is shared in the ratio $1 : \frac{1}{2} : \frac{1}{4}$, what is the smallest part?

A 4 B $4\frac{1}{4}$ C $4\frac{1}{2}$ D $4\frac{3}{4}$ E 5

5. The diagram shows an octagon which is formed by two overlapping rectangles. Which of the following is a correct expression for the perimeter of the octagon?



A $2p + 2r$ B $2p + 2q$ C $2q + 2r$ D $2p - 2q + 2r$ E $2p + 2q - 2r$

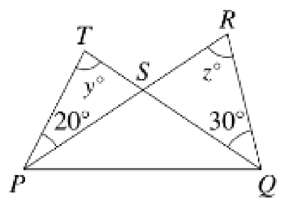
6. The nth term of a sequence is $\frac{n}{n+1}$. What is the first term of the sequence whose value is greater than 0.92?

A $\frac{9}{10}$ B $\frac{10}{11}$ C $\frac{11}{12}$ D $\frac{12}{13}$ E $\frac{13}{14}$

7. The five numbers 95, 96, 97, 98, 99 are each paired with one of the five numbers 1, 3, 5, 7, 9 (each number used once). The larger number of each pair is then divided by the smaller, giving five integer answers.
What is the difference between the smallest and the largest of the five integer answers?

A 96 B 86 C 76 D 66 E 56

8. The diagram shows two triangles PQT and PQR with QT and PR meeting at S. Angle TPR is 20° and angle TQR is 30°.
Angle STP is y° and angle QRS is z°.
What is the relationship between y and z?



A $y = z + 10$ B $y = \frac{3z}{2}$ C $y = z$ D $y = z - 10$ E $y = \frac{2z}{3}$

9. How many of the following expressions are divisible by 24?

$5^2 - 1$	$7^2 - 1$	$9^2 - 1$	$11^2 - 1$	$13^2 - 1$

A 1 B 2 C 3 D 4 E 5

10. A triangle has sides in the ratio $5 : 5 : 6$ and area $108\,\text{cm}^2$.
What is the length, in cm, of the longest side of the triangle?

A 15 B 18 C 21 D 24 E 27

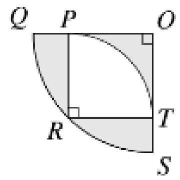
11. In the addition shown, the digits C, I, K, M, T and U represent different digits.

What is the value of $T + I + C + K$?

```
  U K M T
+   I M C
---------
  2 0 2 6
```

A 6 B 15 C 16 D 25
E More information needed

12. Quarter circle PRT is removed from quarter circle OQS to leave three shaded regions, as shown. $OPRT$ is a square and $OP = 2x$.
What is the total area of the three shaded regions?



A $\frac{3x^2}{4}$ B x^2 C $2x^2$ D $\frac{3\pi x^2}{4}$ E πx^2

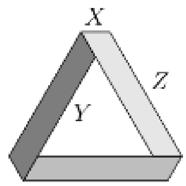
13. Noah writes down the numbers 1, 2, 4 and 13. He then adds a fifth integer x which then becomes the mean, median or mode of the five numbers. How many possible choices for x are there?

A 2 B 3 C 4 D 5 E 6

14. PQR is an acute-angled triangle. Each of its angles, in degrees, is a triangular number.
What is the smallest angle in the triangle?

A 21° B 28° C 36° D 45° E 55°

15. The diagram shows the UKMT logo, which is comprised of three identical parallelograms enclosing an equilateral triangle of side length Y. The ratio of the lengths $X : Y : Z$ is $1 : 4 : 5$.
What is the ratio of the area of the enclosed triangle to the total area of the parallelograms?



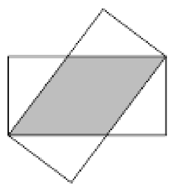
A 1 : 2 B 3 : 5 C 5 : 9 D 8 : 15 E 1 : 1

16. Kerry and Karen are driving to meet each other. They both leave home at mid-day and both drive at the same constant speed. At 1 pm they are 200 km apart by road. At 2:15 pm they are 100 km apart.
How far apart by road do Karen and Kerry live?

A 360 km B 320 km C 280 km D 260 km E 240 km

17. Two identical 1×2 rectangles overlap as in the figure on the right.

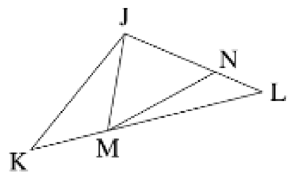
What is the area of the common (shaded) part?



A $\frac{5}{4}$ B $\frac{4}{5}$ C 1 D $\frac{6}{5}$ E $\frac{1 + \sqrt{5}}{2}$

18. In the diagram, $JK = JL$, $JN = JM$ and angle $KJM = 30°$.
What is the value of angle NML?

A $7\frac{1}{2}°$ B $12\frac{1}{2}°$ C $15°$ D $17\frac{1}{2}°$ E $20°$

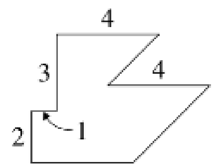


19. For what value of x does $1 + \cfrac{16}{1 + \cfrac{1}{1 + \cfrac{1}{x}}} = 11$?

A $\frac{1}{2}$ B $\frac{3}{5}$ C $\frac{2}{3}$ D $\frac{3}{2}$ E $\frac{5}{3}$

20. All the angles in this figure are multiples of $45°$. Some side lengths are given.
What is the perimeter of the figure?

A $16 + 5\sqrt{2}$ B $18 + 5\sqrt{2}$ C $18 + 4\sqrt{2}$ D $16 + 4\sqrt{2}$
E $20 + 5\sqrt{2}$

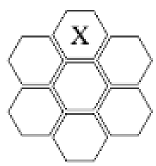


21. The numbers x and y satisfy the equations $x + y = 6$ and $x^3 + y^3 = 198$.
What is the value of xy?

A 1 B 2 C 3 D 4 E 5

22. A bumblebee wants to visit all the hexagons in the diagram shown. He starts at X, only moves between hexagons that share a border, and never visits the same hexagon twice. How many possible routes could the bumblebee take?

A 14 B 16 C 18 D 20 E 24

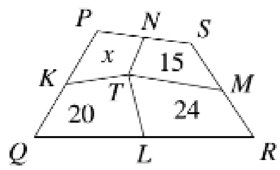


23. Oliver asked a group of people whether they liked Maths. Exactly 99% answered yes. Teresa asked a different group of people whether they liked Maths and combined her results with Oliver's. Now exactly 98% of the people had answered yes. What is the smallest number of people who could have answered no to Teresa?

A 1 B 2 C 3 D 4 E 5

24. The points K, L, M and N are the midpoints of the edges of the quadrilateral $PQRS$, as shown.
T is a point inside the quadrilateral. The quadrilaterals $KQLT$, $LRMT$ and $MSNT$ have areas 20, 24 and 15 respectively.
What is the area of the quadrilateral $NPKT$?

A 10 B 11 C 12 D 13 E 14



25. Zoe has an unusual method of choosing when to go swimming. At the beginning of the year she randomly chooses three different days of the week, then always has a swim on those days only. Using this method, what is the probability her schedule will include at least three days in a row without a swim?

A $\frac{2}{5}$ B $\frac{1}{2}$ C $\frac{3}{5}$ D $\frac{7}{10}$ E $\frac{4}{5}$

